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Contents

CHAPTER 1

The Importance of Being Mathematical

Jane Buerger

Jane Buerger is a graduate of the secondary mathematics program of Concordia University Chicago. After graduation, she taught mathematics at Lutheran High School in Houston and later for the Clear Creek Independent School District and San Jacinto Junior College, also in the Houston area. Dr. Buerger joined the faculty of Concordia College—New York in 1986 as a professor of mathematics and education. During that time she also served as chair of the divisions of science and mathematics and of teacher education. Dr. Buerger earned her master's degree at the University of Houston and her doctorate at Teachers College, Columbia University in New York. In 2005, she returned to her alma mater, Concordia University Chicago, to serve as dean of the College of Education.

Why Do We Have to Do This

The situation is familiar. Math class seems to be going along fairly well; children appear to be catching on to the new concept being taught. There is time for the children to try some new exercises, perhaps similar to what they will be working on later in class or at home. Then a voice is heard from the back of the room. "Why do we have to learn this stuff?" It's a good question, and we, as teachers, should consider why it is being asked before we jump in with an answer.

Why do children ask the question? Do they ask the same thing about their other subjects? Is mathematics somehow different? Is there a good reason for learning how to compute ½ ÷ 4?

One answer that doesn't work very well is any variation of "You'll need to know this someday." "Someday" might be replaced by "next year in sixth grade," "in high school," "to get into college," or "when you're grown up." Children live in the here and now, and it's hard for them to imagine a future when their success will be measured in their ability to do long division. Add to that the fact that, in this country at least, it is socially acceptable to not be "good at math," and the questions that children ask about why they have to learn "this stuff" seem logical.

As teachers, we are responsible for knowing the content that we are teaching. We are also responsible for knowing why our students need to learn that content and then structuring our lessons so that the *why* becomes obvious. We need to design our curricula so that children have a chance to make the connections between their classroom and their world outside of school.

Teaching mathematics is a special challenge. Textbooks are putting more emphasis on having the children solve nonroutine problems, but, in order to be successful at this, children need to master a number of basic skills first. The way to master a skill, whether it is multiplying whole numbers, playing the guitar, or shooting free throws, is practice, practice, practice. For many of us, this was all there was to mathematics. We would learn a new skill, and then we would work pages and pages of exercises. Eventually there would be some word problems, which were really just more exercises in disguise.

Practicing computational skills has a purpose. No responsible mathematics teacher says that children don't need to know their multiplication facts. However, if we never expose children to meaningful situations where being able to multiply (without the help of a calculator) is important, then we are doing a real disservice to them.

So then, how can we help our students see the value of learning "this stuff"? We can structure our lessons and units to help our students develop a sense of how mathematics fits into their world. Following is a list of four reasons why mathematics is important for our students. This is the grown-up version. It will be up to us as the grown-ups to plan lessons that will lead our students to develop their own list of why mathematics is important to them.

Reason 1:

The attitudes and strategies necessary for successful mathematical problem solving carry over into other areas of life.

Mathematical problem solving does not mean working the typical textbook word problems that are really just computational practice in disguise. Even when the textbook authors attempt to be relevant by including references to favorite activities, the truth is that the exercises don't pique the students' interest or give them a real reason for finding a solution. For this discussion, a *problem* will mean a novel situation where the student doesn't have a set rule for approaching it. The student will have to use computational skills in the process, but the procedures will cause the student to develop mathematical thinking and possibly discover mathematical concepts that are new, at least to that student.

A true mathematical problem for some students might be trying to decide if they can earn enough money for some special project, perhaps buying gifts for children in a shelter. George Polya, in his book *How to Solve It*, identified four steps in the problem-solving process. The first step is *understanding the situation*. At this step, we realize that gifts cost money and that, in order to buy the gifts, there must be a way to earn that money.

The second step is *devising a plan*. What do we have to know to solve this problem? We need to know how many children are in the shelter, what type of gifts would be appropriate, and how much these gifts would cost. We need to know what type of fund-raising would be appropriate and would raise the funds we need. We need to decide how we can obtain this information and what we will do with the information when we get it.

The third step is *executing the plan*. We gather all the information about the cost and number of gifts and the amount we could expect to earn.

The fourth step is *looking back*. We need to see if our answers make sense in the context of the problem. If it turns out that we need $1,000 to buy the gifts and our projected fund-raising will result in only $300, then perhaps we need to go back and reexamine our project. Maybe less expensive gifts would be in order; maybe we need to find another way to raise the money.

The point of all of this is that problem solving, in mathematics and in life, must begin with true understanding and careful planning. Too often, students approach mathematical problems by looking for key words, such as *altogether*, and then add every number in the exercise. By allowing students to work on more novel situations, we allow students to take the time to think, to understand, and to look back later to see if their solutions make sense. The procedure won't allow students to solve ten routine word problems for tomorrow's homework, but it will enable them to use mathematical skills as part of a larger process that may actually be practical to them. The procedure will also serve students well as they tackle problems outside of the classroom, whether the problems are rocky relationships or situations involving personal finances or time management.

Reason 2:

Mathematics enhances other subjects in the school curriculum (and vice versa).

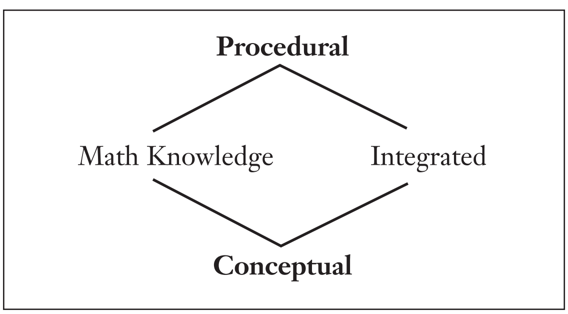
The idea here is to integrate mathematics into the curriculum so that students can actually gain a better understanding of math at the same time they are learning about other subjects. For example, students can gain a better understanding of Hindu-Arabic numerals by studying early numeration systems that did not use place value or zero or that had a base other than ten. A unit on the Roman Empire might include a study of Roman numerals, which could lead to the following questions: What would it be like to add in

CHAPTER 2

Math Is More Than Numbers!

Mathematics is sometimes called the science of patterns. The five areas of mathematics that will be explored in this curriculum guide are **Numbers/Operations**, **Algebra**, **Measurement**, **Geometry**, and **Data Analysis**. The study of math is many things. It is abstract and concrete; it is common and complex; it is theoretical and practical. If you see math as procedures—facts and skills—you are only seeing part of the picture. The procedural aspects of math are the tools that gain in significance when used with the conceptual aspects of math, which involve application, processes, and relationships. A true picture of math knowledge looks at the procedural and conceptual aspects and then integrates them (as illustrated in Diagram 1), rather than isolating them.

Diagram 1



Math relates to so many aspects of daily life. With your students, discuss all the things they would have to do without if they had a day with no applications of math. For example, there would be no telephone (no number pad), no food made from a recipe (no measuring cups), no shopping (no price tags or cash), no sports (no scores or statistics), no weather forecasts (no measuring wind, rainfall, or temperature), no TV channel numbers, and on and on. Math knowledge is more than memorizing facts and drilling procedures. It is a way of thinking that involves logic, patterns, relationships, decision making, problem solving, and communicating applications to concrete life situations. Math is a necessary part of the real world.

Consider also how math relates to many other subject areas you study each day in school: math is crucial to the accuracy of scientific experimentation; in Bible study, we use numbers to locate references in God's Word (such as John 3:16); numbers are significant when studying historical dates, geographic locations (altitude, latitude, populations), sociological and political data, and other aspects of social studies; basic geometry is a fundamental part of creativity in the arts, such as in painting and sculpture; and the relationship of math to musical patterns and notation is inseparable.

Take a close look at Diagram 2, on page 15, which expands on these ideas. The diagram reminds us that the procedural aspects of math (at the center) necessarily should be related to and integrated with all of the conceptual processes in the circles surrounding them. These areas influence each other when math is applied productively and appropriately. To avoid the age-old question students ask about math—*when will we ever use this stuff?*—continually help your students be aware of the fact that math permeates life. Like reading, math is a necessary life skill in our world today. The study of math, therefore, needs to be seen as

Real World
Functional & Authentic
A Valuable Blessing from God

There is more, however, to observe about the valuable blessing of math. Math relates to our daily lives; our faith in Jesus relates to our daily lives. So we need to consider how math and our Christian lives relate to one another, for they do indeed!

One of the most direct connections that children, teenagers, and adults have with math on a daily basis is with the use of money. This is a real-world and vital connection. The Christian implications for this involve Christian ethics and Christian stewardship. (Note that Christian ethics and Christian stewardship are part of the sanctified life we live through the power of the Holy Spirit, having already been completely justified by the grace of God through faith in Christ Jesus, who died on the cross and arose at Easter to give us forgiveness of our sins and eternal salvation.) Real-world discussions and problem solving in math and finance might be one of the best places to teach children about honesty, fairness, and generosity from a Christian point of view. Set up situations (and even act them out to make the math concrete and the drama personal) such as this: *Matthew gave the store clerk $10 to pay for an $8 CD. The clerk, thinking it was a $20 bill, gave Matthew $12 in change. How much profit did Matthew make from this transaction?* Point out in your discussion that the extra $10 he received was not "profit." He was keeping something he knew belonged to someone else. Ask, *What commandment did Matthew break?* (The Seventh Commandment) To encourage further comments, ask, *Would he be foolish to return the money? Why or why not? Where can Matthew get help with this concern?* (God speaks to us in His Word, calls us to repentance, forgives us through Jesus, and guides us through the power of the Holy Spirit to live as people of God.) *If Matthew returned the money, how do you think the clerk might respond? What thoughts might Matthew have after returning the money?*

Another direct and real-world connection between mathematics, Christianity, and our daily lives involves careers—now and in the future. Point out the relevance of math to your students' possible future careers, particularly because most occupations involve a paycheck, and because, today, most careers involve some technology, which usually involves math. At the same time, help students see the relevance of their Christian faith to whatever future careers they may have. Our life as Christians is integral to, not isolated from, all others areas of our lives. Discuss ways to serve God and give Him glory in a variety of occupations. Discuss matters like business ethics, fair trade, and other occupational issues that, as Christians, we look at from the perspective of our sanctified life, having first been justified by grace in Christ Jesus.

There are so many other issues in life where your students will face questions about how to use math, money, time, talents, treasures, and so on for God's glory, to help others, and to wisely use the blessings the Lord has given them. Looking once more at Diagram 2, reading the central line across the illustration, we again are reminded of the interrelatedness of the five areas of math as we connect them to daily life and integrate our faith into all we do.

As a final note, consider that "God's math" is far beyond any equation we may teach or learn in school because it is beyond comprehension!

1 sinner + 1 Savior = 4 giveness

"Great is our Lord, and abundant in power; His understanding is *beyond measure*" (Psalm 147:5, emphasis added).

"God, being rich in mercy, because of the great love with which He loved us, even when we were dead in our trespasses, made us alive together with Christ—by grace you have been saved . . . so that in the coming ages He might show the *immeasurable riches of His grace* in kindness toward us in Christ Jesus" (Ephesians 2:4–5, 7, emphasis added).

CHAPTER 3

An Integrated Approach to Math

So-called math wars have erupted in the teaching of math in recent years between constructivists and back-to-basics advocates. However, more and more educators approve a centrist, balanced approach, seeing this as a matter of both/and rather than either/or. The teaching of math needs micro and macro perspectives. Math education needs to be looked at from several angles, embracing all that is helpful, rather than polarizing into separate camps. The content of effective math instruction includes more than just isolated skills, just as the process and application of math involves more than answering a few story problems tacked on to the end of a chapter. The many aspects of math education need to be integrated.

The four charts in this chapter help us to look at math education comprehensively. Chart 1 lists the five *content* strands of math, giving broad-sweeping generalizations of what math education in all grade levels needs to involve, as developed by the National Council of Teachers of Math (2000) and printed here with their permission (along with Charts 2 and 4). Likewise, Chart 2 gives generalizations of the five *process* strands of math, as developed by the NCTM. Most states in the United States have added dozens of standards per grade level using a similar format. The standards listed in Chapter 4 of this book are based on a compilation of these state standards. As a reminder, however, that these strands and standards cannot serve merely as lists of unrelated skills and processes, we have developed Chart 3, which emphasizes that the many aspects of math must be interrelated, as well as integrated! Chart 3 depicts a well-rounded scenario to be implemented on an annual and daily basis.

The National Council of Teachers of Math realizes that while we need to look at broad generalizations and detailed standards, it is also necessary to have specific focal points. So in 2006, the NCTM developed the focal points listed in Chart 4, giving three key emphases for each grade level in math that serve as the foundation for further study. The NCTM emphasizes that "it is essential that these focal points be addressed in contexts that promote [the processes of] problem solving, reasoning, communication, making connections, and designing and analyzing representations." (For elaboration on each focal point, see the Web site for the National Council of Teachers of Mathematics at www.nctm.org/focalpoints/.)

Within all these perspectives, as educators in Lutheran schools, we want to integrate math into our daily lives, particularly our daily lives as children of God. This is basic to our purpose in Christian education as we thank God for the blessings He provides in this orderly and mathematical world, as we rejoice in the forgiveness and salvation Christ has offered to us, making us His own people through His death and resurrection, and as we are led by the Holy Spirit to live out our lives for the glory of God in all that we do.

Chart 1: Five Content Strands in the Teaching of Math

Numbers and Operations

Instructional programs should enable all students to

- understand numbers, ways of representing numbers, relationships among numbers, and number systems;
- understand meanings of operations and how they relate to one another; and
- compute fluently and make reasonable estimates.

Algebra

Instructional programs should enable all students to

- understand patterns, relationships, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships; and
- analyze change in various contexts.

Measurement

Instructional programs should enable all students to

- understand measurable attributes of objects and the units, systems, and processes of measurement; and
- apply appropriate techniques, tools, and formulas to determine measurements.

Geometry

Instructional programs should enable all students to

- analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- specify locations and describe spatial relationships using coordinate geometry and other representational systems;
- apply transformations and use symmetry to analyze mathematical situations; and
- use visualization, spatial reasoning, and geometric modeling to solve problems.

Data Analysis

Instructional programs should enable all students to

- formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- select and use appropriate statistical methods to analyze data;
- develop and evaluate inferences and predictions that are based on data; and
- understand and apply basic concepts of probability.

CHAPTER 4

Mathematics Curriculum Standards for Students in Grade 8

This chapter includes math standards that have been compiled from the individual state departments of education. They are organized, grade by grade, into the following five areas:

1. Numbers/Operations
2. Algebra
3. Measurement
4. Geometry
5. Data Analysis

The Concordia standards have been systematized according to the following numerical designations to indicate grade level, area, and performance objective:

- The first digit indicates the grade level (e.g., the *8* in *8.3.2* designates that the performance expectation is for grade 8).
- The second digit indicates the area of math, as listed above, addressed by the standard (e.g., the *3* in *8.3.2* designates that the standard is in the area of Measurement).
- The third digit indicates the number of the specific performance expectation. These expectations will vary from level to level (e.g., the *2* in *8.3.2*, as found in the Measurement area of the grade 8 standards refers to the second item in that area).

Chapter 5 provides faith-integration activities organized by category. These activities provide many opportunities to teach aspects of the Christian faith in conjunction with each area of the math curriculum. Each activity is keyed to a specific performance expectation.

A complete list of math standards performance expectations for this grade level is provided on the remaining pages of this chapter. The delineation between grade 7 and grade 8 sometimes blurs, so you will need to interchange standards between these two levels to comply with your proposed curriculum and your state standards.

Numbers/Operations

8.1 Eighth-grade students will develop knowledge about numbers and their related operations, increase in computational skill, and explore using a growing numerical sense in real-life situations.

8.1.1 Read, write, compare, and solve problems using large and small numbers in scientific notation.

8.1.2 Recognize natural numbers, whole numbers, integers, rational numbers, and irrational numbers and their relation to the set of real numbers.

8.1.3 Describe the effects of multiplication and division on integers.

8.1.4 Evaluate negative integer exponents. Interpret positive integer powers as repeated multiplication and negative integer powers as repeated division or multiplication by the multiplicative inverse.

8.1.5 Apply order of operations to simplify expressions and perform computations involving integers, exponents, and radicals.

8.1.6 Explain and use the inverse and identity properties and use inverse relationships in problem-solving situations.

8.1.7 Determine when an estimate is sufficient and when an exact answer is necessary in problem situations.

8.1.8 Compare and order rational numbers and percents. Know that every rational number is terminating or repeating and that every irrational number is non-repeating.

8.1.9 Understand that computations with an irrational number and a rational number (other than zero) produce an irrational number.

8.1.10 Estimate, compute, and solve problems involving rational numbers, including ratio, proportion, and percent. Evaluate the reasonableness of solutions.

8.1.11 Find the square root of perfect squares. Place non-perfect squares on an integer number line. Use, explain, and simplify fractional exponents.

8.1.12 Solve problems by computing simple and compound interest.

8.1.13 Use mental techniques to compute with common fractions, decimals, powers, and percents.

8.1.14 Demonstrate an understanding that use of a calculator requires appropriate mathematical reasoning and does not replace the need for mental computation.

C H A P T E R 5

Information and Activities for Integrating the Faith as Keyed to Grade 8 Standards

The math standards included in this chapter have been compiled from the individual state departments of education and organized, grade by grade, into the following five areas:

1. Numbers/Operations
2. Algebra
3. Measurement
4. Geometry
5. Data Analysis

The Concordia standards have been systematized according to the following numerical designations to indicate grade level, area, and performance objective as described on the first page of chapter 4.

Performance expectations are numbered sequentially (e.g., *8.3.2* is found in grade 8, relating to the area of Measurement, and is the second item in that area). A complete list of math standards performance expectations for this grade level is provided in chapter 4.

On the pages of chapter 5, you will find an easy-to-reference two-column format for faith integration with the math standards. The left-hand column under the heading “Information by Topic” provides helpful teaching background information and insights relevant for integrating some aspect of the Christian faith. The number following the topic identifies the performance expectation to which the topic relates (see chapter 4). Beside each entry, in the right-hand column under the heading “Discussion Points/Activities,” you will find ideas helpful for planning and organizing student learning experiences that reinforce and expand upon these faith connections.

Be sure to consult the index at the end of this volume for a complete listing of topics and where they may be found.

INFORMATION BY TOPIC

DISCUSSION POINTS/ACTIVITIES

8.1 Eighth-grade students will develop knowledge about numbers and their related operations, increase in computational skill, and explore using a growing numerical sense in real-life situations.

Numbers in Scientific Notation.

We need scientific notation because God created such an amazing world. We live in a world where things are "really big" and "really small." God made a universe so expansive that we have not even begun to explore and understand it, but at the same time He designed us on a molecular level that requires specialized equipment just to begin to study it. God is not bound by size and space. No number is big enough to quantify Him. (8.1.1)

• In a total eclipse, the orb of the moon just covers the orb of the sun. This situation can occur because the moon and the sun have the same apparent diameter visible from Earth, even though the sun is much larger. This is evidence of God's perfect design. Because the sun's diameter is approximately 4×10^2 times larger than the moon's and the sun is approximately 4×10^2 farther away, from the Earth, the moon appears to be as large as the sun. The numbers involved in this situation are large numbers. Convert each measurement into scientific notation.

a. Distance from moon to Earth: 239,000 mi (2.39×10^5)

b. Moon's diameter: 2,159 mi (2.159×10^3)

c. Distance from sun to Earth: 93,000,000 mi (9.3×10^7)

(Note that the Earth travels around the sun in an ellipse, so this measurement is an average.)

d. Sun's diameter: 868,000 mi (8.68×10^5)

• To explore the relationship between the sun and the moon, multiply the moon's diameter and distance by 4×10^2 and compare with the sun's measurements. While these numbers are not exact, they are remarkably close. When you look at these intricacies of God's massive creation, what kind of picture of Him do you see? Contrast this picture of God with the picture shown in Matthew 6:28–30.

• Why do you think God wanted to design His universe in a way that would include eclipses? Read Matthew 27:45. Some speculate that this might have been an eclipse. If so, how does this make the creation of the world more awesome? (At creation, God set the world in motion in a way that would cause an eclipse in Palestine at that time.) How did God use this darkness to make a statement? (Sin brings an eternal darkness that was eclipsed by Jesus' sacrifice on the cross.)

INFORMATION BY TOPIC | DISCUSSION POINTS/ACTIVITIES

Relating Real Numbers to Other Numbers

An important mathematical idea is the completeness of the real numbers. There are no gaps within this set of numbers. It may seem like an obvious idea, but when you really look at the complexity of numbers such as the square root of 2, we can see that it is a beautiful system. As our God created this universe, He did so completely. We are able to see this in the mathematics that came out of His creation and also in His creation itself. After God finished each step, He observed that "it was good." God made a perfect world for us, complete with a perfect plan for our salvation. (8.1.2)

• The number 1.61803399... is often called phi, the golden ratio, or the divine proportion. It is a number that appears frequently in nature, in art, and even in the shape of credit cards. You can read more about phi in many books on mathematics or on the Internet. There are several places in the Bible where the golden ratio (or an approximation of it) pops up. Can you find it in Genesis 6:15 and Exodus 25:10? (The ratios of the width to the height of Noah's ark, whether in feet [$75/45$] or cubits [$50/30$], as well the ratio of the length to the width or height of the ark of the covenant [$2.5/1.5$] all equal 1.6666….) Do you think God put it there on purpose? (Note that 1.6666... is the closest that you can come to phi with simple numbers.) Fun numbers appear often in our world. Sometimes people are distracted by them, thinking that the numbers themselves are what's important. It's good for us to remember that our faith is based in Christ and simply to enjoy the interesting patterns for what they are worth.
• Sometimes God seems to use numbers to mean something more than their numeric value. Can you think of any examples where this is the case? (7 as a number of completeness: creation in 7 days, Peter must forgive 70 × 7 times; 40 as a number of testing or trial: 40 days of flood, Israel in the desert 40 years, Jesus fasted 40 days.)
• God and His love for us cannot be bound by any numbers—real or otherwise. How does John 10:30 demonstrate this fact? (Jesus + the Father = one God.) What number mystery do you find when you compare Matthew 28:19 with 1 Corinthians 8:4? (Father + Son + Holy Spirit = one God.)

Multiplication and Division of Integers

At first glance, the multiplication and division of integers presents itself as just one simple rule to follow: remember whether the answer is positive or negative. Teachers are often tempted to say, "This is easy," and students are tempted simply to memorize the rules. In truth, the concept of multiplication and division of integers is a very abstract idea and complicated to understand. When someone says, "This is easy," and you do not agree, it

• In the Bible, Jesus multiplies with integers during His ministry. Read Mark 6:30–44. In this story, the math Jesus does goes something like this: 7 (loaves and fish) × 1 (no additional objects; simply words of thanks) = 5,000 (at least the amount of loaves and fish to feed the people). It seems like Jesus even had the power to change the rules of mathematics.
• Read Acts 2:14, 41. What strange mathematical calculation could you write for the day of Pentecost? (1 × 1 = 3,000; 1 preacher × 1 sermon = 3,000 new

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